while from Eqs (3 8)

$$G_2(Q_f\ \dot{n}_f\ n_f)=0 \tag{13}$$

Equations (12) and (13) form a system of nonlinear dif ferential equations When $h(t)$ is known, it can be numerically integrated with appropriate initial conditions If we only wish to seek the steady state solutions, the initial conditions can be chosen arbitrarily To simplify the procedure, we define nondimensional quantities such as

$$\xi=4Q_f/(\pi^2 d^3 n_f) \tag{14}$$

$$\psi=\psi(\xi)=2P_f/(\rho\pi^2 d^2 n_f^2) \tag{15}$$

$$\lambda_f=\lambda_f(\xi)=16T_f/(\rho\pi^3 d^5 n_f^2) \tag{16}$$

$$\lambda_p=\lambda_p(n_f)=16T_p/(\rho\pi^3 V^{5/3} n_p^2) \tag{17}$$

where $V$ is a characteristic volume of the powerplant and $d$ the diameter of the fan

Substitution of Eqs (14) to (17) into Eqs (12) and (13) leads to

$$g_1(\dot{n}_f\ n_f\ \dot{\xi}\ \xi;\ \dot{h}\ h)=0 \tag{18}$$

$$g_2(\dot{n}_f\ n_f\ \xi)=0 \tag{19}$$

where

$$g_1=\psi-\frac{Ld}{2Sn_f^2}\xi\dot{n}_f-\frac{Ld}{2Sn_f}\dot{\xi}-\left[\frac{\zeta}{2S_f^2}+\frac{1}{2(cC_dh)^2}\right]\frac{\pi^2d^4}{8}\xi^2$$
$$+\frac{Sh}{(cC_dh)^2}\frac{d}{2n_f}\xi-\frac{1}{\pi^2d^2n_f^2}\left(\frac{S\dot{h}}{cC_dh}\right)^2 \tag{20}$$

$$g_2=\frac{16(J_f+\kappa^2\eta_tJ_p)}{\rho\pi^3d^5}\frac{\dot{n}_f}{n_f^2}-\kappa^3\eta_t\frac{V^{5/3}}{d^5}\lambda_p+\lambda_f \tag{21}$$

Equations (18) and (19) were solved using the fourth order Runge Kutta method with intervals of 0 001 s in $t$ $\psi(\xi)$ and $\lambda_f(\xi)$ were approximated by polynomials of seventh and third order in $\xi$ respectively, using the method of least squares $\lambda_p(n_f)$ was similarly approximated by a polynomial of the third order in $n_f$ A simple harmonic oscillation

$$h=h_e+\Delta h\ \sin(2\pi t/T) \tag{22}$$

is assumed To confirm the independence of the steady state solutions of the initial conditions computation was made for several cases with the same $T$ and $\Delta h/h_e$ but with different initial conditions The results have been compared with the previous linear[7] and nonlinear[8] quasisteady solutions and the experimental results [8] Results are shown in Fig 1 A sketch of the experimental setup is shown in Fig 2 The plenum oscillates horizontally and the instantaneous cushion pressure and the hoverheight are measured electrically The details of the experiment and of the method of analysis are discussed in Ref 8 Experiments were made for $T\gtrsim 1$ s and $\Delta h/h_e \lesssim 0$ 8 The mean hoverheight $h_e$ was kept at 0 984 in (25 mm) throughout As $T$ decreases (and as $\Delta h/h_e$ increases) the discrepancy between the quasisteady solution and the ex perimental results become noticeable [8] The present nonlinear unsteady solution gives good results A quasisteady con sideration of the compressibility of air was also made using the method employed by Yano and Nagayama [4] In the present study the difference has been so small that it cannot be clearly represented graphically It can almost be concluded that the inertance of air in the duct is the dominant unsteady effect in the present study

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# An Inverse Solution for Component Positioning Using Homogeneous Coordinate Transformations


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## Introduction

HOMOGENEOUS coordinate transformations are used widely in computer graphics and aircraft computer aided conceptual design [1 3] They are used to create or thographic and perspective views and also can be used to allow local axis systems for the separate components which comprise the aircraft three dimensional data base In such an application each component's local axis system is defined by six parameters; i e , $x_l$ $y_l$ $z_l$ (origin offset), and roll, pitch and yaw (axis orientation) Matrix operations are used to transform the points or equations which describe the com ponent into the global axis system These matrix operators are the 3 × 3 direction cosine relationships which are a subset of the homogeneous coordinate transformations


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A problem frequently encountered is the inverse solution; i e  given a desired component orientation, what are the required $x_1$ $y_1$ $z_1$, roll, pitch, and yaw? Typical examples include orienting a strut to span two points, rotating a skewed component about some point in the global axis system, and rotating a component about an offset trunnion axis

This Note presents a solution to these types of problems based upon the application of direction cosines to the homogeneous transformation matrix  This method allows calculation of the $x_1$ $y_1$ $z_1$ roll, pitch, and yaw of a com ponent's local axis system given the global locations of three points; namely, the local axis origin, a point on the local $X$ axis, and a point on the local $X$  $Y$ plane

Figure 1 depicts the homogeneous coordinate trans formation geometry  The global axis system is designated $X_g$ $Y_g$ $Z_g$  The typical aircraft component shown is defined in a local axis system ($X$  $Y$  $Z$ ), and is positioned by six values; the local axis origin point $P_1$ (i e , $x_1$ $y_1$ $z_1$) and the rotation terms; i e , roll ($\gamma$), pitch ($\alpha$) and yaw ($\beta$), which are defined about the local axis origin point ($P_1$)  Two other points shown will be employed later; $P_2$, a point on the local $X'$ axis, and $P_3$, a point on the local $X$  $Y$ plane

To obtain global axis system values ($x_g$ $y_g$ $z_g$) from the local axis system points ($x$  $y'$  $z$ ), the points are rotated by postmultiplication of three $3\times3$ direction cosine matrices (defined in the references), and then the local axis system origin offset ($x_1$ $y_1$ $z_1$) is added to the resulting values  Note that these rotations occur about the translated axis system ($X_t$ $Y_t$ $Z_t$) which is parallel to the global system, not about the final local system ($X$ , $Y$  $Z$ )  Thus, the order of rotation affects the final positioning, and therefore must be used consistently

## Inverse Solution

The inverse problem occurs when the desired component position and orientation are known, but the required $x_1$ $y_1$ $z_1$, roll, pitch, and yaw are unknown  If the desired com ponent position and orientation are known, then the global system positions of points $P_1$ $P_2$ and $P_3$ in Fig 1 can be determined  From these, this solution proceeds by con struction of a rotational matrix based upon the three points $P_1$ $P_2$ and $P_3$ as given in Eqs (1 15)  These are derived in Ref 4  Note that $P_1$ must be subtracted from the three points ($P_1$ $P_2$ and $P_3$) to get a pure rotational problem prior to calculation of the $L$ $M$ and $N$ terms.

$$\begin{bmatrix} L_X & L_Y & L_Z \\ M_X & M_Y & M_Z \\ N_X & N_Y & N_Z \end{bmatrix} \tag{1}$$

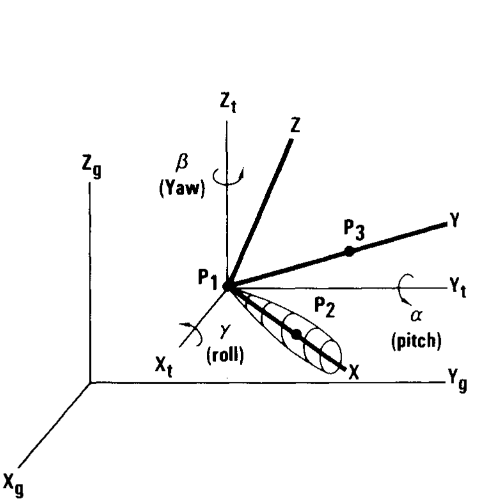


Fig 1  Homogeneous coordinate transformation geometry

$$L_X = \frac{x_2 - x_1}{\Delta} \tag{2}$$

$$L_Y = \frac{y_2 - y_1}{\Delta} \tag{3}$$

$$L_Z = \frac{z_2 - z_1}{\Delta} \tag{4}$$

$$\Delta = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{5}$$

$$N_X = N'_X / \delta \tag{6}$$

$$N_Y = N'_Y / \delta \tag{7}$$

$$N_Z = N'_Z / \delta \tag{8}$$

$$N'_X = (y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1) \tag{9}$$

$$N'_Y = (z_2 - z_1)(x_3 - x_1) - (z_3 - z_1)(x_2 - x_1) \tag{10}$$

$$N'_Z = (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1) \tag{11}$$

$$\delta = \sqrt{N'^2_X + N'^2_Y + N'^2_Z} \tag{12}$$

$$M_X = N_Y L_Z - N_Z L_Y \tag{13}$$

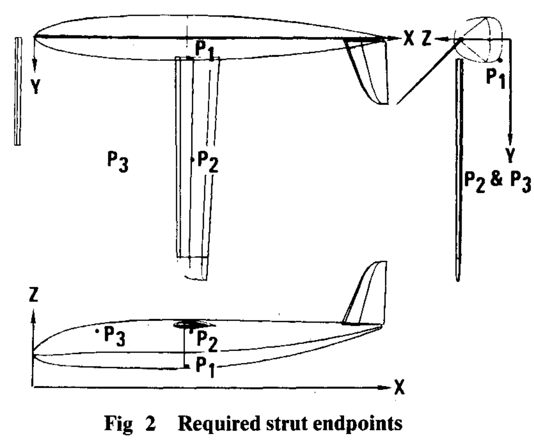


Fig 2  Required strut endpoints

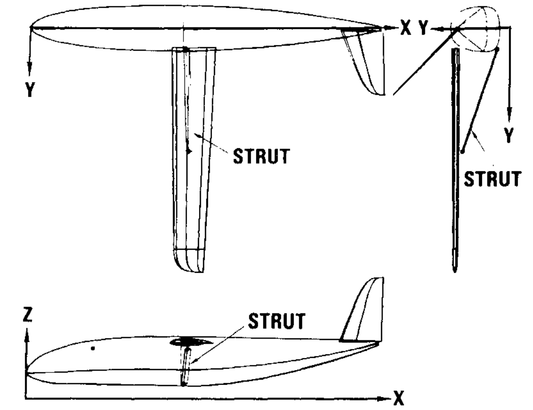


Fig 3  Resulting strut location

$$M_Y = N_Z L_X - N_X L_Z \quad (14)$$

$$M_Z = N_X L_Y - N_Y L_X \quad (15)$$

Next an equivalent $R$ matrix using homogeneous coordinate transformation equations is developed by combining the three rotational matrices in the references As mentioned previously, the order of combination is important If they are combined as roll times pitch and then times yaw, the following matrix is formed:

$$\begin{bmatrix} [\cos\alpha \cos\beta] & [\cos\alpha \sin\beta] & [-\sin\alpha] \\ [(\sin\gamma \sin\alpha \cos\beta) - \cos\gamma \sin\beta] & [(\sin\gamma \sin\alpha \sin\beta) + \cos\gamma \cos\beta] & [\sin\gamma \cos\alpha] \\ [(\cos\gamma \sin\alpha \cos\beta) + \sin\gamma \sin\beta] & [(\cos\gamma \sin\alpha \sin\beta) - \sin\gamma \cos\beta] & [\cos\gamma \cos\alpha] \end{bmatrix} \quad (16)$$

Equating terms to the matrix in Eq (1) gives nine equations in three unknown variables From these we can determine that

$$\alpha = \sin^{-1}(-L_Z) \quad (17)$$

$$\beta = \cos^{-1}(L_X/\cos\alpha) = \sin^{-1}(L_y/\cos\alpha) \quad (18)$$

$$\gamma = \sin^{-1}(M_Z/\cos\alpha) = \cos^{-1}(N_z/\cos\alpha) \quad (19)$$

Either of the two possible values of $\alpha$ are correct $\beta$ and $\gamma$ are uniquely determined once $\alpha$ is selected

The required $x_l$ $y_l$ and $z_l$ local axis origin values are the coordinates of $P_1$, which were subtracted from the coor dinates of $P_2$ and $P_3$ to get a pure rotation problem prior to calculation of the $L$ $M$ and $N$ terms

There is a trivial case to consider If $\cos\alpha = 0$, there is no solution by this method This occurs when the local $X$ axis aligns with the global $Z$ axis This is indicated when $|L_Z| = 1$ and $L_x = L_y = 0$ thus $\alpha = (-L_z)\pi/2$

As we have selected rotation order of roll, then pitch, then yaw it follows that when pitch is $(-\pi/2)$, the roll and yaw must sum to the total angle $\theta$ between the local $Y$ axis and the global $Y$ axis If the pitch is $+\pi/2$ the yaw has negative sign, thus

$$\beta - L_z \gamma = \theta \quad (20)$$

where

$$\theta = \sin^{-1}(-M_x) = \cos^{-1}(M_y) \quad (21)$$

Any combination of $\beta$ and $\gamma$ meeting the equations is correct

## Application of the Inverse Procedure

We have now identified a procedure for determining $x_l$ $y_l$ $z_l$ roll, pitch, and yaw for a local axis system given three points; the local axis system origin $P_1$ a point $P_2$ on the desired local $X'$ axis and a point $P_3$ on the desired local $X$ $Y'$ plane

Applications of this procedure now will be described The most direct application is to allow the designer to specify that a component will go from "here to there" by input of three points The first point is "here" and becomes $P_1$ in the preceding calculations The second point is "there" and becomes $P_2$ The third point, $P_3$, specifies only the com ponent roll about the $X$ axis by defining the $X'$ $Y$ plane This capability is shown in Figs 2 and 3

Figure 2 shows a typical design problem A wing strut is required extending from point $P_1$ to point $P_2$, and is oriented such that the airfoil streamwise direction ($Y$ axis) is ap proximately parallel to the global $X$ axis $P_3$ is coincident with $P_2$ in rear view and forward of $P_2$ in top view thus orienting the $X$ $Y$ plane properly Note the initial strut location which is irrelevant to our solution

Figure 3 shows the resulting strut location The calculated values obtained in this example are $x_l = 70\ 35$ $y_l = 9\ 07$, $z_l = -9\ 28$, roll = 0 7 deg, pitch = 18 8 deg and yaw = 87 9 deg

A second utilization of this procedure for determining the local axis system values permits component rotations in the global axis system regardless of the original orientation of the local axis system

This can be implemented using the preceding equations by a simple trick We take three arbitrary points in the local axis system, convert them to global coordinates rotate them in the global system, and then use the given procedure to find the new local axis system $x_l$ $y_l$ $z_l$, roll pitch, and yaw To simplify matters the three local axis system points selected are $P_1 = (0, 0\ 0)$, $P_2 = (1, 0, 0)$, and $P_3 = (0, 1\ 0)$

Trunnion axis rotations can be implemented in a similar fashion This and the related problem of trunnion axis location are detailed in Ref 4

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# A Method for Measuring Skin Friction Drag on a Flat Plate in Contaminated Gas Flows


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## Introduction

THE most straightforward way to measure drag on a surface immersed in a fluid flow is by direct measurement of the force on the exposed surface [1] This usually involves replacing a portion of the surface with an imbedded sensor surface generally about 0.5 to 1 in in diameter (some noncircular sensors have been used) The surface is directly connected to a small force transducer beneath the plate It is possible to achieve good results with these sensors but care must be taken in their installation[2] to avoid misalignment of the surfaces and binding between the sensor surface and the surrounding plate surface

An alternate method of determining surface drag is through indirect methods based on similarity arguments and/or cer

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